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A Suspension: Using Conic Sections in Persian Arch Structures

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Abstract

Different views exist about using conic sections in historical arch structures; this issue is present in Persian architecture. This paper explores the possibility of such a function in the pre-Islamic and Islamic architecture of Iran. To examine this question, the history of conic sections in Iran and its neigh boring civilizations is considered here, and experts' attitudes towards the matter are discussed. The results show that there is no historical document that proves this use. Moreover, the attitudes of experts who believe in this use are incorrect or biased. It seems our theories affect our observations and analysis and makes them biased.

Keywords: Arch geometry; conic sections; Persian architecture

1. Introduction

Different approaches have been employed to study the geometry of Persian architecture: transcendental interpretations of it (see e.g. (Ardalān; Bakhtiār, 1975) and many others), investigation of geometrical drawing methods and aspects of practicality (see e.g. (Besenval 1984) and many others), investigation of its structural capability (see e.g. (Pirnia 2008) and many others), and.... This paper considers a topic in geometrical drawing methods and aspects of practicality: whether conic sections have been used to construct arches in Persian architecture or not. A look at the history of Persian mathematics, and criticism of experts' attitudes, are the approaches taken here to answer the paper's query. There is a problem in some geometrical studies of Persian architecture, probably due to a positivist view: experts sometimes ascribe their interpretations to the work of art, without any evidence confirming that their interpretations are meaningful for the local people or for those in the past, and without any evidence from historical documents. The problem can be seen in some aspects of the study of Persian architecture, which we should take into account in this paper. We need documentary evidence, which is sometimes forgotten. If we assume that using conic sections is meaningful in the cultural context, then we can consider it a phenomenon only because it exists or try to find the reasons behind it. Secondly, we need to know whether there is any advantage to investigating the use of conic sections in arch structures. This issue can be looked at from two perspectives: architectural authenticity, and structural advantages. As regards the first issue, we need to consider certain points. Some experts have employed conic sections to redraw arches for restoration, though clearly this negligent practice can cause problems with architectural authenticity. Then, the structural benefits of using conic sections can be examined. In literature about structural capability of arches both the conic sections and catenary curve have been considered. Golombek et al (1988) have quoted from Bulatov (1988) that the structural capability of pre-Islamic elliptical arch inspired Islamic master builders to continue this geometrical method. Huerta (2007:214) confirming Choisy (1904) says that the trajectory of compressions (the line of thrust or inverted catenary) is the origin of the oval arches. However, the structural (rational) interpretation of a masonry arch is a matter of debate and should be carried out in depth.

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For instance, when the 3-centered arch in pre-Islamic Iran has less horizontal thrust in comparison to the *Panj-o-haft* Savafid arch, the master builder employed the arch with more thrust in Safavid era (Fig.1): Hanlon (2006) notes that cultural issue of using arch is also important. Accordingly, the author considers the matter only to clear a historical problem.

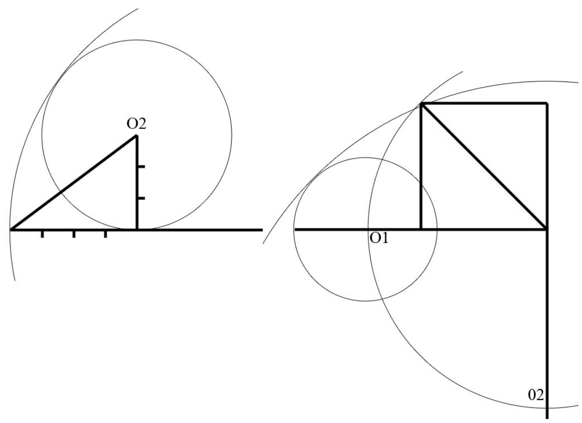

Fig. 1: Left: Drawing Method of a pre-Islamic Arch in Iran (Ghyka(1977:23). Right: drawing Method of *Panj-o-haft* arch (Pirnia 2008: 103). Safavid arch has less Thrust (Drawn by author)

Practical use of conic section, in the Islamic era, has been noted by experts such as Özdural (2000), Necipoglo (1995) and…, e.g. in drawing *gereh* which is based on historical documents. However, Hogendijk (1998), noting some uses of conic sections in Islamic era, e.g. for making astrolabe, says that they are not of such practical value. Arch, vault and dome drawing methods in Islamic and Persian architecture have been studied by many experts (see (Dold-Samplonius, 2000), (Golombek et al, 1988), and many others). Their studies on historical documents, for instance, on the well-known calculation by Kāshāni, show compassed-base drawing for arches but not using conic sections. Only in recent decades, using the conic sections has been noted in some books (See Bulatov (1988), Zomarshidi (1983), Abu al-Qāsemi (2005), and Pirnia (2012)). Different attitudes exist regarding the use of conic sections in arch structures worldwide. Huerta (2007: 217) strongly disagrees with it and regards such arches as ovals, criticizing Daressy (1907) and Arnold (1991) (for more explanation see (Huerta 2007, 215)). Also, a part of this discussion is about Choisy`s (1904) analysis on Egyptian arches and recognizing them as ovals. However, as Huerta (2007, 217) says "… one sees what one wants to see", there is a discussion about how to compare arches to mathematical curves: We need historical documents.

In Persian architecture studies, however, some experts have declared that there is no doubt about use of the conics. As an introduction to the study of arch structures, Besenval (1984) has investigated conic sections. Zomarshidi (1983, 128, 310), Abu al-Qāsemi (2005, 90), and Pirnia (2012, 330) believe in the use of parabola and ellipse in pre-Islamic architecture. Pirnia (2012: 330) has also noted a tool for constructing a dome using conic sections that we shall look at later in this paper. Moreover, Necipoglo (1995, 191), Pirnia (2012, 330), and Abu al-Qāsemi (2005, 91), Taheri (2009) have suggested their use in Islamic era. Some experts, such as Zomarshidi (1983, 128), have used conic sections to redraw arches: they might even be used for restoration or design. In short, it seems that certain experts in Persian architecture have assumed the matter to be obvious, though none of them has produced persuasive evidence from historical documents, or (in this case) interviews with *ustāds* (master architects or builders). Only Necipoglo (1995, 190-196) has presented some historical documents as proof of the positive view. We face some concerns in this instance: are these opinions and reasons correct? Is there any historical evidence for using conic sections to draw arches in Persian architecture? The author investigates the history of conic sections from before the Greek mathematicians to the end of the Islamic era and architectural documents in Islamic Iran in order to find possible indications to answer the query. Then, the author discusses the attitudes of the experts who believe the answer is positive.

2. History of Conic Sections in Iran and its Neighboring Regions

2.1. Mathematical Documents

Here we take a quick look at the history of the conic section in Iran and its neighbors. These investigations are carried out in three well-known historical periods as follows (see (Pāshā et al. 2006: 26-36) and (Shahriāri 2006, 17)):

- Primary period (before 7th century BCE)
- Development of mathematics by Greeks (7th century BCE to 9th century CE)
- Development of mathematics by Persian mathematicians (9th to 15th century CE).

Primary Period (before 7th Century BCE)

Historians of mathematics have not noted anything concerning the study of the conic section at this time in Iran and Mesopotamia (see (Eves 1990)). Drawings similar to the conic section have been found in Egypt. Certain experts, like Huerta (2007, 217), do not believe in using the ellipse in such arches and strongly disagree with Daressy (1907) and Arnold (1991). They use geometrical drawing with compasses to investigate the issue. However, they have not found any historical documents as proof of their position, so in this case the matter is vague. In the history of mathematics, India was well-known among Persians due to its arithmetic, not for its geometrical tradition. This can be verified by books translated into Arabic in Iran during the Islamic period, such as *Rāshikāt al-Hind* (Biruni 2010). These books show that Indian mathematicians did not influence geometry in Iran despite the close relationship between these two countries. Iran was not greatly affected by Indian geometry and Indians were not well known in conic section studies, so the possibility of collaboration in this field cannot be considered. Based on available documents, it does not seem that the Chinese used conic sections in ancient times. Needham (1959, 102) believes that the first discussion in China about employing conic sections dates to the 17th century CE. As we shall see in this paper, Persian mathematicians were aware of conic sections much sooner than the Chinese. We can conclude that no evidence has so far pointed to the use of conic sections in the above ancient civilizations.

Development of Conic Sections by the Greeks

A short summary of Greeks` work in the matter is provided in Table 1. As the Persian mathematicians continued and developed the Greeks` works, not considering this relationship has made some biased interpretation as we shall see.

Table 1: Chronological Summary of Greeks Conic Section Studies

From Thales (7th-6th centuries BCE) to Pythagoras (50 years after Thales
There is no evidence about the issue (Heath 1921a, 1-170).
From Pythagoras to Plato (5th century BCE),
The Pythagorean method concerns the application of areas. This is important as Apollonius used this technique to describe the principles of conic sections (Heath 1921a, 150).
From Plato (4th century BCE) to Euclid (3rd-4th century BCE)
Menaechmus (350-360 BCE)
The discovery of conic sections has been attributed to from the period, (Heath 1921b, 110).
From Euclid (3rd-4th century BCE) to Pappus (3rd century CE)
Archimedes
-determining the area under the curve parabola and parabolic segment, as well as the volume of hyperboloid of revolution (3rd century BCE) (Heath 1921b, 27-28). - He also mentioned the burning mirror (Heath 1921b: 18).
Apollonius of Perga (Pāshā et al. 2006, 60).
-The major evolution in the study of conic sections
Pappus
-He provided formulas for finding volume and area according to center of gravity for a solid of revolution (Tanton 2005, 372). - He also solved the problem construction of a conic section through five points (Heath 1921b, 434). -Trisection of an angle was in addition one of the tasks for which he proposed suitable methods (Heath 1921a, 235).

There is a crucial question here because of Greek geometry's influence on Persian mathematics in the following era. Did the Greeks use conic sections to draw arches? It seems that the Greeks used them only for their intellectual satisfaction. In order to solve the numerical problem of the geometric mean for two numbers, Menaechmus considered conic sections but he was not thinking about any practical problem. It is not known why he decided to cut a cone (Heath 1921a, 110).

It is mentioned that a Greek architect of the 6th century CE had discussed elliptical and parabolic mirrors (Heath 1921b, 541) and had presented an architectural dilemma that was solved using conic sections (Heat h 1921b, 541). He had also introduced the ellipse drawing method (Heath 1921b, 542). There is, however, no document to prove the use of conic sections in arch structures in Greece. Further, Greeks were not interested in using and developing arch structures because it was not culturally meaningful to them (Hanlon, 2006, 70). They used arches with semi-circular forms (Huerta, 2007, 217): However, it was for secondary buildings such as sewers ((Boyd 1978), Dornisch, 1992)). Huerta (2007, 217) disagrees with using ellipse systematically in Greece and Rome. The Greeks not using conic sections suspends the probability of their employment in Persian architecture, which had a role in the next phase of conic section evolution.

Conic Sections in Islamic Iran (9th-15th Centuries CE)

Researches on the conic section conducted by Persian mathematicians in the Islamic era have been recorded over time, as shown in Table 2. (Dold-Samplonius (2003) has discussed some of them and, here, the present author gives further works in this field.)

Table 2: Chronological Summary of Iranian Conic Section Studies in Islamic era

1 – Thābit Ibn Qorra (826-901CE)
– Greek mathematicians' books (Al-Magest etc.) were translated into Arabic by Thābit Ibn Qorra or under his supervision. (Thābit Ibn Qorra and Haelal Hamsi translated the book Conics by Apollonius of Perga) (Qorbāni, 1998, 204).– He discussed determination of the area under the curve parabola and the volume of the paraboloid (Qorbāni, 1998, 205). – Ketāb fi Shekl al-Molaqqab Bi al-Aqta` (Book on the shapes known as (conic) sections) (Qorbāni 1998, 206)) – Fi Masahat al-Ashkāl al-Mosattaheh va al-Mojassameh (On the area of two- and three-dimensional shapes) (Qorbāni, 1998, 206) – Makhrotāt-e Apollonius (The conics of Apollonius). This book was first translated by Haelal Hamsi (Qorbāni, 1998, 204). – Masahat al-Mojasamāt al-Mokāfieh (The surface area of the paraboloid) (Qorbāni, 1998, 207).
2 – Banu Musā (several brothers from Khorasan) (9th century CE)
– Tahrir-e Makhrutāt-e Abolonious (Rewriting the Conics of Apollonius) (Qorbāni, 1998, 151).
3- Māhāni (9th century CE)
– Solving equations using conic sections (Pāshā et. al., 2006, 102). – In Resalah fi Qesmateh Rob al-Dāyereh (Book on division of a quarter circle), Khayyam states that Māhāni tried to solve a problem introduced by Archimedes, but Abu Jafar Khāzen did so (Rezāzādeh Malek, 1998, 49).
4 – Sejzi (945-1026 CE)
– The trisection of an angle problem, which had been solved with a moving ruler, was elucidated by Sejzi by intersecting a circle and equilateral hyperbola. He named this the fixed geometry method (Pāshā et al. 2006, 90). He wrote treatises about conics (Pāshā et al., 2006, 90). – Treatise on asymptote of hyperbola (Qorbāni, 1998, 258). – Resalah fi Khavās-e Shekel al-Mojassam al-Hādeth men Edārat al-Qat' Zā'ed va al-Mokāfi (Book on the characteristics of the hyperboloid and paraboloid) (Qorbāni, 1998, 255). – Fi al-Khavās al-Qobbat Zā'ed va al-Mokāfi (Book on the characteristics of paraboloid and hyperboloid domes) (Qorbāni, 1998, 255). – Fi Khavās al-Mojassam al-Nāqeseh va al-Za'edeh va al-Mokāfieh (Book on the characteristics of the volume of paraboloid, ellipsoid and hyperboloid) (Qorbāni, 1998, 255). – Resalah fi Vasf Qotu' al-Makhrotiah ((Book describing conic sections) (Qorbāni, 1998, 255).
5– Ibn Haytham (965-1040 CE)
– In his book Al-Manāzar (Optics and perspective), he solved a problem in optics (a quadratic equation by intersection of a hyperbola and a circle) (Qorbāni 1998, 48). – Resalah fi Masāhat al-Mojassamah al-Mokāfiah (Book on determining the area of a paraboloid) (Qorbāni, 1998, 48).
6– Biruni (937-1048 CE)
–Biruni used conic sections to solve the trisection of an angle problem (Pāshā et. al., 2006, 21).
7 – Abual-Jud (10th-11th century CE)
– He solved the equation $x^3 + a = cx^2$ using conic sections (Qorbāni 1998, 70). – He also changed another geometrical problem to a quadratic equation and solved it by intersection of a parabola and an equilateral hyperbola (Qorbāni, 1998, 70).
8 – Kuhi (940-1000 CE)

– Resalahfi al-Parkār al-Tām va al-Amal Bih (Book on the perfect compass and its use) (Pāshā et. al., 2006, 101) – Resalah fi Estekhrāj Masāhat al-Mojassam al-Mokāfi (Book on finding the surface area of a paraboloid) (Qorbāni, 1998, 425).
9 – Khayyam (1048-1131)
– According to Rezāzādeh Malek (1998, 50), Khayyam presented 10 of the 21 famous equations of the Islamic era. Ibn-e Khaldon (1987, 1013) states that some great men in the east (Khayyam) increased the number of equations from 6 to 20.
10 – Abu Nasr-e Iraq (960-1036)
– He addressed three problems concerning the perfect compass in his book Resalah fi al-Javāb an Baʿz Masāel al-Hendeseh (Book of answers to some geometrical problems), which he wrote in response to Biruni (Qorbāni, 1998, 117).
11 – Abu al-Fath-e Isfahani (1060-?)
– He summarized the conics sections of Apollonius of Perga's book Conics (Qorbāni, 1998, 93).
12 – Kamal al-Din Yunos (ca. 1050)
– With Mohammad Ibn-e Hossein, he wrote the book Al-Barkār al-Tām (Perfect compasses) (Qorbāni, 1998, 398).
13 – Abd al-Malek Shirazi (before 1155-1203)
– Summary of seven papers on conics sections from Apollonius of Perga's book Conics (Qorbāni, 1998, 305).
14 – Mohammad Ibn-e Hossein (11th century)
– Resalah al-Barkār al-Tām va Keyfiat al-Takhtit Bih (Book of perfect compasses and how to use them to draw) (Qorbāni, 1998, 446).
15 – Athir al-Dir al-Abhari (11th century)
– Resalah fi Barkār al-Qotuʿ (Book on perfect compasses) (Pāshā et. al., 2006). – Sharh-e Kitāb-e Abolonius fi al-Makhrutat (Description of conics sections from Apollonius of Perga's Conics book) (Qorbāni, 1998, 461).

At the same time (Islamic era), some studies were conducted in the West, but there is no evidence regarding their effect on Persian mathematicians. Kepler (15th-16th century) worked on some problems using conic sections (Field, 1997, 183-185). Waliss was one of the first experts to consider a conic section as a quadratic equation curve, not as a section of a cone (Maor, 2013, 51). According to Qorbāni (1998, 4), in 1694, for the first time, a Persian mathematician discussed new mathematical studies in the West. This is known as the end of the Islamic era. In Iranian historical encyclopaedias, definitions of conic sections and their applications are provided; however, there is no reference to their use in arch structures (see e.g. (Biruni 2007: 26-28)). Based on historical investigations, Persian mathematicians continued, developed, and made practical those studies about the problems the Greeks had started to work on, though there is no evidence in mathematical documents to confirm the use of conic sections in arch structures. Hogendijk (1998) believes the Islamic mathematicians considered conic section as a matter in pure mathematics: Even in mathematical drawing, investigating the problems of the conic sections, they used arcs of circle instead of drawing the conic sections (Hogendijk, 1998).

2.2. Architectural documents in Islamic Iran

Few documents exist concerning Persian architectural drawings, especially from the time before the Mongol conquests of Iran. Some architecture-related drawings are found in the works of Persian mathematicians, such as Buzjāni (940-998) (2005) and Kāshāni (1380-1429) (1987); however, they contain nothing about using conic sections in arch structures. Some scholars believe that mathematicians redrew conic sections in a simple way for practical use by master architects or builders (Necipoglo, 1995, 191) but there is no evidence to prove this in the case of arches. Taheri (2009) says that "…while knowing that elliptical profiles are in use in arches and domes of Islamic architecture and circular profiles are not suitable arches for curved coverings (Dold-Samplonius, 2003, 247), it is surprising that he (Kāshāni) has not even considered the ellipses separately."; however, he has not noted based on what historical document he assumes this use is obvious: Whether not considering the matter by Kāshāni shows that master builders did not employ ellipse. Necipoglo (1995, 190-197) introduces certain documents to demonstrate the possibility of the use of conic sections in Islamic architecture. One of her notes directly considers that the arch and others indicate the probable use of conic sections in architecture. She mentions a book that we have already mentioned in this paper: *Resalah fi Estekhrāj-e Masāhat al-Qobbat al-Mokāfieh*, by Thābit Ibn Qorra (Necipoglo, 1993, 190-197). She states that the calculation for obtaining the area of a paraboloid dome (*qobbat al-mokāfieh*) can be considered as proof of using conic sections in domes. This interpretation is biased, however. The word *qobbeh* "dome", which was used in this book, is derived from the Persian word *gonbad*, employed by Islamic scientists and even in Persian literature for all hemispherical shapes similar to a dome, not only for a dome in a building. Moreover, Thābit Ibn Qorra`s book discusses Archemidous's problem, which is not related to architectural domes.

Musavi Bojnordi (2007, 748-749) has also identified these works of Thābit as studies on Archemidous`s problems. The document is not related to architecture. A note by Necipoglo (1993, 191) points to the use of a tool for drawing conic sections. Obviously this can only demonstrate their use, for example, in *gereh* drawings not arches. She also mentions the lost book of Ibn Haytham, known as *Maqālah fi Ejārat al-Hozzur va al-Abnieh be Jami' al-Ashkāl al-Hendsieh*, which had a section on conics at the end (Necipoglo 1995, 190). It does not demonstrate anything about arches: We should consider that some mathematical documents involve both practical and pure problems. She (Necipoglo 1995, 190) considers the Ibn-e Khaldon book a historical document about using conic sections. According to Ibn-e Khaldon (1987, 1017) conic sections were used to build *hayākel-e Mo'azameh* (great buildings). He states that conic sections are used in carpentry, masonry work, sculptures, special buildings, and the master builders pulling heavy loads (Ibn-e Khaldon, 1987, 1017). It is not clear whether he was referring to *Elm al-Esqāl* (statics), *Elm al-Heyal* (mechanics), or geometry. Ibn-e Khaldon was a historian, so it is not certain how much he knew about how to use geometry in architecture. His work cannot be regarded as documentary proof in this issue. Accordingly, the first document note by Necipoglo (1995, 190-197) cannot be regarded as evidence for the use of conic sections in arches. Other documents do not consider arch geometry. Pirnia (2008) has mentioned some examples of the use of conic sections in Persian arches. First, he (2008, 103) has noted that some pointed arches are in fact intersections of two ellipses, demonstrating this view in a drawing (Fig. 2); however, author's drawing shows the arch geometry and Pirnia's method are different. Golombek et al (1988) have also quoted this method from Bolatov, again without any historical documentation. Critics of this interpretation stated that pointed arches were common in Assyria when conic sections were not known so this idea cannot be understood chronologically. As historical evidence Kāshāni`s (1987) account of arches and dome geometry does not involve conic sections. Further, drawing an intersection of two ellipses is more difficult than drawing arcs in situ.

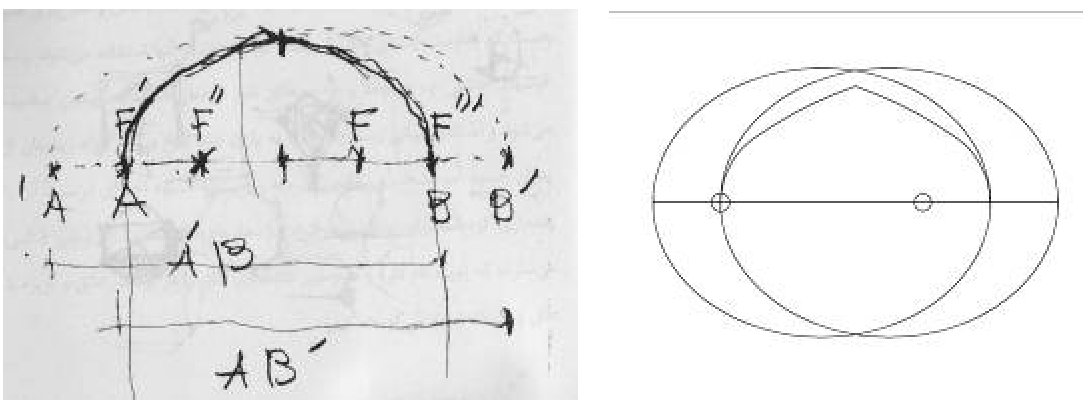

Fig. 2: Left: A drawing by Pirnia, who Considered the Pointed Arch as the Intersection of two Ellipses (Pirnia 2008: 103). Right: Author's drawing shows the Difference between arch Geometry and Pirnia's Method

Second, Pirnia[i] (2008, 95) and Abu al-Qāsemi (2005, 91) have viewed some pre-Islamic and Islamic domes as conic sections and suggested a way to construct elliptical domes. According to their explanation, the master architects or builders position a wooden bar vertically in the center of their dome plan. Then they construct the geometry of the dome using two nails on the bar and a chain (Fig. 3). According to Abu al-Qāsemi (2005, 91) this tool was known as a *Shāhang-o Hanjār*. Any person with experience of drawing an ellipse using two foci on paper knows how difficult it is for master builders to control a metal chain in, for example, a 10 meter span: the chain will bend. It should be added that Pirnia (2008: 95) and Abu al-Qāsemi (2005, 91) have not presented any historical documents or even the names of master architects or builders, from whom they are quoting this method. Pirnia (2012, 330) only noted that this tool remained in the Rahim Khān mosque in Isfahan[ii]. Looking at this tool in this mosque (Fig. 4), it is obvious that this is not a tool for drawing ellipse – the dome is pointed, not an ellipse, according to documented map (see (Hāji-qāsemi 1996, 118))![iii]

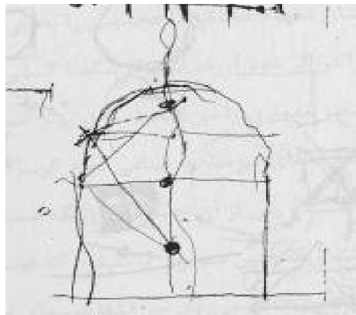

Fig. 3: Pirnia's drawing Of the Tool *Shāhang-O Hanjār* for Constructing Elliptical Domes (Pirnia 2008:103).

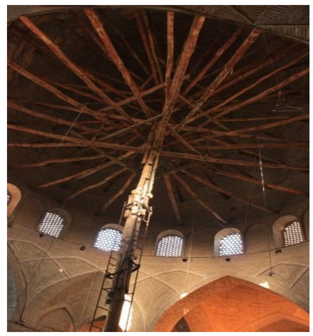

Fig. 4: Rahim Khān Mosque in Isfahan. It is Obvious that this is not a Tool for Drawing ellipse. (Photographs by: H. Golchin)

In conclusion, there is nothing to commend the view of experts who believe this to be a tool for drawing ellipses. Some experts, such as Jazbi (in description of the book of Buzjāni (2005)), have noted the drawing method for elliptical arch, with two concentric circles, by Persian master builders; however, without presenting a historical document or interview with master builders. This method was firstly proposed by Serlio (1545) in the Renaissance. However, Serlio did not know that it was an ellipse (Huerta, 2007, 230). Golombek et al (1988) have also quoted using this method in Islamic era from Bolatuv, again without any historical document. Another problem in construction related to probable use of ellipse, is making the groin in groined vault (Fig. 5). Huerta (2007, 219) discusses the matter, considering both "lengthened arch" and the construction without the physical building of the diagonal centring. In Parisian architecture, *tāq-e Chahār-bakhsh*, groined vault has been less used than other kinds of vaults and domes: Considering that Persian architecture is mostly based on brick rather than stone, perhaps, the difficulty of constructing the groins with brick is a reason for this problem. (Heyman (1966, 266) has also noted the importance of the construction of groins). However, Persian master builders used centering for two vaults and find the groin physically. Sometimes, master builders built groin which is a rare case; however, they used an arc that goes through three points.

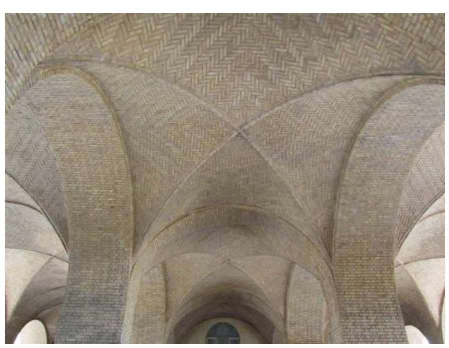

Fig. 5: *Arg* Square in Kerman. Master Builders Built Groin using an arc that goes Through Three Points (Photograph by the Author)

In recent documents, *Ustad* Lorzādeh has noted the drawing method of ellipse for arch. Because he has been a master builder, his note can be considered, but we should consider that he has lived in recent 100 years. He might be affected by new mathematics.

3. Discussion

Based on historical investigations, Persian mathematicians continued, developed, and made practical those studies on conic section problems that the Greeks had started. In this case, there is no evidence in mathematics documents to confirm the use of conic sections in Iranian arch structures. From previous studies, there is nothing to confirm that conic sections were used in arch structures. In Besenval's work (1984), there is no consistent and documentary relationship between his introduction about conic sections and the arches he has investigated. Moreover, the notes of Taheri (2009) and Zomarshidi (1983) are not documentary. The tool for dome construction noted by Pirnia (2008) is also not documented. Neither can the case he introduced from the Rahim Khān mosque be accepted. Necipoglo`s note (1995) about historical documents not acceptable. In short, we can see that their views are not persuasive. In the author's opinion, the study of conic sections can constitute a biased approach, influenced by similar Western studies in the 18th century. Have researchers in Persian architecture adopted a Western attitude to Persian architecture? The author believes that this is in fact possible. It seems as Kuhn says, our theories affect our observations and analysis and makes them biased (Kuhn, 1962).

4. Conclusion

The possibility of the use of conic sections in Persian architecture has been considered in the present paper and the following outcomes can be reported:

- Investigation of this question in Iran, in relation to the neighboring civilizations, demonstrates that despite crucial developments of this type in Iran, there is no sign of the use of conic sections in arch structures. This work was done to develop Greek studies to solve some well-known problems.
- A discussion about the attitudes of experts to the matter in hand shows that their views are biased. No evidence exists in historical documents to settle the question.

Finally, the results demonstrate that the use of conic sections in Persian architecture needs to be confirmed by historical documents.

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Notes

[2] It is worth mentioning that Pirnia contradicted the attitude to Persian architecture of a Western researcher (such as Godard). He also left Tehran University to protest the social and cultural context of academic architecture education in Iran in the 1940s. This matter inspired him to find counterparts in Iran who were against the western approach, e.g. introducing the Golden Sectionto the country. However, in the author's opinion, some of his ideas are not documented.

[3] Some experts, such as Pirnia, categorized Sasanian and Ashkanid under one, Parthian architectural style.

Bibliography

ABU AL-QĀSEMI, Latif, (2005), The Islamic Art and Architecture of Iran, Tehran: Omran va Behsāzi-e Shahri.

ARDALĀN, Nāder and Lāleh BAKHTIĀR, (1975), The Sense of Unity: The Sufi Tradition in Persian Architecture, Chicago: University of Chicago Press.

ARNOLD, Dieter, (1991), Building in Egypt, Pharaonic Stone Masonry, Oxford: Oxford University Press.

BESENVAL, Roland, (1984), Technologie de la voûte dans l'Orient Ancien. 2 vols, Paris: Editions Recherche sur les Civilisations.

BIRUNI, Abu Reyhān, (2010), Rāshikāt al-Hind, Kāveh Izadi, ed. and tr, Tehran: Miras-e Maktoob Publisher.

____________________, (2007), Al-Tafhim, Jalalal-Din Homaee, ed. and tr., Tehran: Homa Publisher.

BULATOV, (1988), Mitkhat Sagadatdunovich, Geumetricheskaia Garmonizatsiia v Arkhitekture Sredmnei Azii IX-XV (Geometric Harmonization in the Architecture of Central Asia from the Ninth to the Fifteenth Century), Rev. ed., Moscow: Izdatel`stvo "Nauka".

BOYD, Thomas D., (1978), 'The arch and the vault in Greek architecture', American Journal of Archaeology, 82: 83-100, 1978.

BUZJĀNI, (2005), Hendes-e (Applied geometry), S.A. Jazbi, tr. Tehran: Soroush Press.

CHOISY, Auguste, (1904), L'art de bâtir chez les égyptiens, Paris: E. Rouveyre, 1904.

DARESSY, George, (1907), ‘Une trace Egyptienne conservée d’unevoûte elliptique. Annales du service des antiquités de l’Egypte’, **8** (1907), 234-241.

DOLD-SAMPLONIUS, Yvonne, (2003), ‘Calculating Surface Areas and Volumes in Islamic Architecture’, The Enterprise of Science in Islam New Perspectives, edited by Jan P. Hogendijk and Abdelhamid I. Sabra, Massachusetts: The MIT Press,.

________________, (2000), ‘Calculation of Arches and Domes in 15th Century Samarkand’, Nexus III: Architecture and Mathematics, ed. Kim Williams, Pisa: Pacini Editore, 2000, 45-55.

DORNISCH, Klaus, (1992), Die griechischen Bogentore, Zur Entstehung und Verbreitung des griechischen Keilsteingewölbes, Frankfurt am Main: Peter Lang.

EVES, Howard, (1990), An Introduction to History of Mathematics, The Saunders Series, Saunders College Publications.

FIELD, Judith Veronica, (1997), The Invention of Infinity: Mathematics and Art in the Renaissance, Oxford: Oxford University Press.

GHYKA, Matila, (1977), The Geometry of Art and Life, New York: Dover Publications, INC.

GOLOMBEK, Lisa, D. WILBER et al., (1988), The Timurid Architecture of Iran Turan, Vol. 1, Princeton: Princeton University Press.

HĀJI-QĀSEMI, ed., (1996), Ganjnameh (Cyclopaedia of Iranian Islamic Architecture), vol. 2. Mosques of Isfahan, Tehran: Faculty of Architecture and Urban Planning Documentation and Research Center.

HANLON, Don, (2006), ‘Arches and Culture’, Nexus Network Journal, **8**(2006) 67-72.

HEATH, Thomas Little, (1921a) ‘A History of Greek Mathematics, vol.1, Oxford: Clarendon Press.

_______________,(1921b), A History of Greek Mathematics, vol. 2, Oxford: Clarendon Press.

HEYMAN, J., (1966), ‘The stone skeleton. International Journal of Solids and Structures’, 2 (1966), 249-279.

HUERTA, S., (2007), ‘Oval Domes: History, Geometry and Mechanics ’, Nexus Network Journal, **9** (2007), 211-248.

IBN-e KHALDON, (1987), Maghaddame-ye Ebn-e Khaldon, Abdolrahman and Parvin Gonabadi Mohammadi, ed. and tr., Tehran: Elmi Farhangi Publisher.

KĀSHĀNI, Gh., (1987), Resāle-ie Tāq-o Azaj, Tehran: Soroush Press.

MUSAVI BOJNORDI, ed., (2007), The Encyclopaedia of Islam, vol. 16. Tehran: Centre for the Great Islamic Encyclopaedia.

NECIPOGLO, Gulru, (1995), The Topkapi Scroll-Geometry and Ornament in Islamic Architecture With an Essay on the Geometry of the Muqrnas by Mohammad Alasad, Los Angeles: The Getty Centre of the History of Art and Humanities.

NEEDHAM, Joseph, (1959), Science and Civilization in China, vol. 3 , Cambridge: Cambridge University Press.

PĀSHĀ, Einoljah et al.,.(2006), Farhang-e Riaziat (Dictionary of mathematics), Tehran: Madreseh.

PIRNIA, Mohammad Karim, (2003), Sabk-Shenāsi Miʿmāri-e Iran, Q. Meʿmārian, ed., Tehran: Pajuhandeh Publisher.

_______________, (2008), Tahqiq dar Miʿmāri Gozashte-ye Iran, Q. Meʿmārian, ed, Tehran: Soroush Panesh.

______________, (2012), ‘Gonbad dar Miʿmari-e Iran ’, Z. Bozorgmehri, ed. Athar, 20, 24 (2012) 330.

QORBĀNI, Abu al-Qāsem, (1996), Biographie des mathematiciens de l’époque islamique, Tehran: Nashr-e Daneshgahi Press.

REZĀZĀDEH Malek. R., (1998), Omar-e Khayyam, Tehran: Sedāi-e Moʿāser and Elm-o Honar.

SERLIO, Sabastiano, (1545), Il Primo libro d’Architettura di Sebastiano Serlio. Paris.

TAHERI, Jafar, (2009), Mathematical Knowledge of Architecture in the Works of Kâshânî, NEXUS NETWORK JOURNAL – VOL. 11, NO. 1 (2009),77-87

TANTON, James Stuart, (2005), Encyclopaedia of Mathematics. Infobase Publishing, 2005.

ZOMARSHIDI, Hossein, (1983), Tagh va Ghos dar Memari-e Iran (Vault and arch in Persian architecture), Tehran: Keyhan Publisher, 1983.

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[i] He noted that this tool has been preserved for further study but no one has considered it as a case study (Pirnia 2008: 95).

[ii] He has noted that this tool has saved for further studies [Pirnia 2008:95] but no one has considered it as a case studies

[iii] Furthermore, it is noteworthy to mention that Pirnia contradicted some western researchers such as Godard’s attitude towards the Persian architecture. He also left the Tehran University to protest the social and cultural context in academic architecture education in the 1940s in Iran. This matter inspired him to find counterparts in Iran against western attitudes e.g. introducing special golden-section in Iran. However, in the author`s opinion, some of them are not documentary.